Quantitative Literacy:
Thinking Between the Lines

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Chapter 4:
Personal Finance

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Lesson Plan

- ▸ Saving money: The power of compounding

- ▸ Borrowing: How much car can you afford?

- ▸ Savings for the long term: Build that nest egg

- ▸ Credit cards: Paying off consumer debt

- ▸ Inflation, taxes, and stocks: Managing your money

Learning Objectives:

▸ Calculate a balance after *t* deposits

▸ Calculate needed deposit to achieve a financial goal

▸ Determine the value of nest eggs (an annuity)

▸ Determine a monthly annuity yield

▸ Determine an annuity yield goal (the nest egg needed)

▸ **Example:** You deposit $100 to your savings account at the end of each month and suppose the account pays a monthly rate of 1% on the balance in the account. Find the balance at the end of three months.

▸ **Solution:**

  ▸ At the end of 1st month:

  New balance = Deposit = $100

  ▸ At the end of 2nd month:

  New balance = Previous balance + Interest + Deposit
  = $100 + (1% × $100 = $1) + $100 = $201

  ▸ At the end of 3rd month:

  New balance = Previous balance + Interest + Deposit
  = $201 + (1% × $201 = $2.01) + $100 = $303.01

4.3 Saving for the long term: Build that nest egg

TABLE 4.2 Regular Deposits into an Account

At end of month number	Interest paid on previous balance	Deposit	Balance
1	$0.00	$100	$100.00
2	1% of $100.00 = $1.00	$100	$201.00
3	1% of $201.00 = $2.01	$100	$303.01
4	1% of $303.01 = $3.03	$100	$406.04
5	1% of $406.04 = $4.06	$100	$510.10
6	1% of $510.10 = $5.10	$100	$615.20
7	1% of $615.20 = $6.15	$100	$721.35
8	1% of $721.35 = $7.21	$100	$828.56
9	1% of $828.56 = $8.29	$100	$936.85
10	1% of $936.85 = $9.37	$100	$1046.22

▸ **Regular deposits balance:** regular deposits at the end of each period.

> ## Regular deposit formula
>
> $$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1 + r)^t - 1)}{r}$$

▸ **Example:** Suppose we have a savings account earning 7% APR. We deposit $20 to the account at the end of each month for five years. What is the account balance after five years?

▸ **Solution:** The monthly interest rate $r = \dfrac{APR}{12} = \dfrac{0.07}{12}$. The
number of deposits $t = 5 \times 12 = 60$. The regular deposit
formula gives:

$$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1 + r)^t - 1)}{r}$$

$$= \frac{\$20 \times \left(\left(1 + \frac{0.07}{12}\right)^{60} - 1\right)}{\frac{0.07}{12}} = \$1431.86$$

▸ The future value is $1431.86.

▸ Determining the savings needed

$$\textbf{Deposit needed formula}$$

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

▸ **Example (Saving for college):** How much does your younger brother need to deposit each month into a savings account that pays 7.2% APR in order to have $10,000 when he starts college in five years?

▸ **Solution:** We want to achieve a goal of $10,000 in five years.

The monthly interest rate $r = \dfrac{APR}{12} = \dfrac{0.072}{12} = 0.006.$

The number of deposits $t = 5 \times 12 = 60.$ The deposit needed formula gives:

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

$$= \frac{\$10000 \times 0.006}{((1 + 0.006)^{60} - 1)} = \$138.96.$$

He needs to deposit $138.96 each month.

Chapter 4 Personal Finance
4.3 Saving for the long term: Build that nest egg

▸ **Example (Saving for retirement):** Suppose that you'd like to retire in 40 years and you want to have a future value of $500,000 in a savings account, and suppose that your employer makes regular monthly deposits into your retirement account. If your expect an APR of 9% for your account, how much do you need your employer to deposit each month?

▸ **Solution:** Goal = $500,000, $t = 40 \times 12 = 480$

r = Monthly rate = $\dfrac{APR}{12} = \dfrac{0.09}{12} = 0.0075$

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

$$= \frac{\$500{,}000 \times 0.0075}{((1+0.0075)^{480} - 1)} = \$106.81$$

▸ **Example (cont.):** Assume the interest rate is constant over the period in question. Over a period of 40 years interest rates can vary widely. Assume a constant APR of 6% for your retirement account. How much do you need your employer to deposit each month under this assumption?

▸ **Solution:** r = Monthly rate = $\frac{APR}{12} = \frac{0.06}{12} = 0.005$

$$\text{Needed Deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

$$= \frac{\$500{,}000 \times 0.005}{((1+0.005)^{480} - 1)} = \$251.07$$

Note t   hat the
monthly deposit more than doubles.

▸ **Nest egg:** the balance of your retirement account at the time of retirement.

▸ **Monthly yield:** the amount you can withdraw from your retirement account each month.

▸ An **Annuity:** an arrangement that withdraws both principal and interest from your nest egg.

Annuity Yield Formula

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times r \times (1+r)^t}{((1+r)^t - 1)}$$

▸ **Example:** Suppose we have a nest egg of $800,000 with an APR of 6% compounded monthly. Find the monthly yield for a 20-year annuity.

▸ **Solution:** $r = \dfrac{APR}{12} = \dfrac{0.06}{12} = 0.005, \quad t = 20 \times 12 = 240$ months.

Monthly annuity yield

$$= \frac{\text{Nest egg} \times r \times (1 + r)^t}{((1 + r)^t - 1)}$$

$$= \frac{\$800,000 \times 0.005 \times (1 + 0.005)^{240}}{((1 + 0.005)^{240} - 1)}$$

$$= \$5731.45$$

4.3 Saving for the long term: Build that nest egg

Annuity Yield Goal

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

▸ **Example:** Suppose our retirement account pays 5% APR compounded monthly. What size nest egg do we need in order to retire with 20-year annuity that yields $4000 per month?

▸ **Solution:** $r = \dfrac{APR}{12} = \dfrac{0.05}{12}, \quad t = 20 \times 12 = 240$ months

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

$$= \frac{\$4000 \times ((1 + 0.05/12)^{240} - 1)}{(0.05/12 \times (1 + 0.05/12)^{240})} = \$606{,}101.25$$

Chapter 4 Personal Finance: **Chapter Summary**

▶ **Savings:** simple interest or compound interest

  ▶ Formulas: simple interest earned

   period interest rate

   balance after t periods

   APY

   Present value or Future value

   Number of periods to double

▶ **Borrowing:** an installment loan

  ▶ Formulas: Monthly payment

   Amount borrowed

  ▶ Fixed-rate mortgage vs. ARM

Chapter 4 Personal Finance: **Chapter Summary**

‣ Saving for the long term: Build the nest egg (Annuity)
  ‣ Formulas: Balance after t deposits

    Needed deposit

    Monthly annuity yield

    Nest egg needed

‣ Credit cards
  ‣ Formulas: Amount subject to finance charges

    Balance after t minimum payments

‣ Inflation, taxes, and stocks
  ‣ Understand CPI, taxes, DJIA